# ANALYSE DES DONNEES COMBINEES SLAC-BCDMS-NMC A GRAND x: $\alpha_{\rm s}$ ET "HIGH TWISTS".

### S.I. ALEKHIN

l'Institute de la Physique des Hautes Energies, Protvino, 142284, Russie

Nous avons fait une analyse en QCD, a NLO l'approximation, de la combinaison des data de cross-sections profondes non-elastiques SLAC-BCDMS-NMC avec determination de  $\alpha_s$ . Nous montrons que la valeur obtenue pour  $\alpha_s$  depend de la procedure statistique du traitement des erreurs systematiques. L'ajustement des donnees avec la prise en compte complete des correlations point-a-point donne une valeur  $\alpha_s(M_Z) = 0.1183 \pm 0.0021(exp.) \pm 0.0013(echellederen.)$  compatible avec les mesures de LEP et la moyenne mondiale. Nous avons extrait d'une facon independante des modeles les contributions "high twist" aux fonctions de structures  $F_L$  et  $F_2$ : la contribution de "twist-4" a  $F_L$  est en qualitatif accord avec les predictions du modele "renormalon" infrarouge; celle de "twist-6" a  $F_L$  tend faiblement vers les valeurs negatives et celle de "twist-6" a  $F_2$  vers les valeurs positives, quoiqu'elles soient toutes les deux compatibles avec zero compte tenu des erreurs.

## COMBINED ANALYSIS OF SLAC-BCDMS-NMC DATA AT HIGH x: $\alpha_{\rm s}$ AND HIGH TWISTS

#### S.I. ALEKHIN

Institute for High Energy Physics, Protvino, 142284, Russia

We perform a NLO QCD analysis of the nonsinglet part of the combined SLAC-BCDMS-NMC data on inclusive deep inelastic cross section with the extraction of  $\alpha_s$ . We show that the value of  $\alpha_s$  obtained in the analysis is sensitive to the statistical inference procedures dealing with systematic errors on the data. The fit with the complete account of point-to-point correlations of the data gives the value of  $\alpha_s(M_Z) = 0.1183 \pm 0.0021(exp.) \pm 0.0013(ren.scale)$  that is compatible with the LEP measurements and the world average. Model independent x-shape of high twist contributions to the structure functions  $F_L$  and  $F_2$  is extracted. Twist-4 contribution to  $F_L$  is found to be in qualitative agreement with the predictions of infrared renormalon model. Twist-6 contribution to  $F_L$  exhibit weak trend to negative values, and twist-6 contribution to  $F_2$  - to positive values, although both are compatible with zero within errors.

Data on deep inelastic scattering (DIS) of charged leptons off fixed targets  $^{1,2,3}$  are an unique source of information about nucleon structure and value of strong coupling constant  $\alpha_s$ . These data are obtained using high integral luminosity samples and their typical statistical errors are O(1%). However the experimental uncertainties are dominated by systematic errors that typically are 2-3 times as statistical ones. Systematic errors are more difficult to be accounted since they are correlated and an estimators which involve correlated data are complicated. This is the reason, why in many analysis systematic errors are accounted using simplified approach and/or partially (or even completely) ignored. The aim of our present study is to perform QCD analysis of the data from Refs.<sup>1,2,3</sup> with a particular attention to thorough account of point-to-point correlations due to systematic errors. A reliable estimate of  $\alpha_s$  implies the study of possible influence of high twist (HT) contribution to the scaling violation. In our analysis we perform simultaneous and model independent extraction of HT contribution to  $F_L$  and  $F_2$  and study their correlations with the  $\alpha_s$  value.

To allow for extraction of  $F_L$  we analyzed the data on cross sections separated by the beam energies instead of merged data on  $F_2$ . We imposed a cut of  $x \ge 0.3$  to prevent additional uncertainties due to a poorly known gluon distribution. This cut leaves data that to a good approximation can be described by the pure nonsinglet structure functions, which essentially reduces the number of fitted parameters. The cut  $x \le 0.75$ , omitting the region where the binding effects in deuterium are large was also imposed in the analysis. The total number of the data points (NDP) left after the cut is 1243; the  $Q^2$  range of the data is 1 GeV<sup>2</sup>  $< Q^2 < 230$  GeV<sup>2</sup>; the total number of independent systematic errors is 47.

The QCD input leading-twist (LT) structure functions of the proton and neutron were

parametrized at the starting value of  $Q_0^2 = 9 \text{ GeV}^2$  as follows: <sup>a</sup>

$$F_2^{p,n}(x,Q_0) = A_{p,n} x^{a_{p,n}} (1-x)^{b_{p,n}}$$

and then were evolved over the region of  $Q^2$  occupied by the data in NLO QCD approximation in the  $\overline{MS}$  factorization scheme <sup>5</sup> with the help of the code used earlier. The final formula for the structure functions used in the fit, with account of the twist-4 contribution was chosen in an additive form

$$F_{2,L}^{(p,d),HT}(x,Q) = F_{2,L}^{(p,d),TMC}(x,Q) + H_{2,L}^{(p,d)}(x) \frac{1 \text{ GeV}^2}{Q^2},$$

where  $F_{2,L}^{(p,d),TMC}(x,Q)$  are given by NLO QCD with the account of target-mass correction. The functions  $H_{2,L}^{(p,d)}(x)$  were parametrized in a model-independent way: their values at x=0.3,0.4,0,5,0.6,0.7,0.8 were fitted; between these points the functions were linearly interpolated. The data on differential cross sections were fitted using the formula

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2(s-M^2)}{Q^4} \left[ \left(1 - y - \frac{(Mxy)^2}{Q^2}\right) F_2^{HT} + \left(1 - 2\frac{m_l^2}{Q^2}\right) \frac{y^2}{2} \left(F_2^{HT} - F_L^{HT}\right) \right],$$

where s is total c.m.s. energy,  $m_l$  is scattered lepton mass and y is lepton scattering variable.

To take into account the point-to-point correlations of the data points we minimized a functional

$$\chi^{2} = \sum_{K,i,j} (f_{i}/\xi_{K} - y_{i}) E_{ij} (f_{j}/\xi_{K} - y_{j}),$$

where K runs through the data subsets obtained by separation of all analyzed data on experiments and targets, and i through data points within these subsets. The matrix  $E_{ij}$  is the inverse of the covariance matrix  $C_{ij} = \delta_{ij}\sigma_i\sigma_j + f_if_j(\vec{s}_i^K \cdot \vec{s}_j^K)$ , and each vector  $\vec{s}_i^K$  includes all independent systematic errors for the K-th data subset. The other notations are:  $y_i =$  the measurements;  $\sigma_i =$  the statistical errors;  $f_i =$  the theoretical model prediction depending on the fitted parameters. The normalization factors  $\xi$  were fitted for old SLAC experiments and fixed at 1 for BCDMS, NMC, and SLAC-E-140.

The results of this fit are given in Fig. 1. In view of large errors of  $H_L^P(x)$  we imposed the constraint  $H_L^P(x) = H_L^D(x)$  in the final fit. The statistical quality of the fit is acceptable:  $\chi^2/\text{NDP} = 1255/1243$ . The value of strong coupling constant obtained is  $\alpha_s(M_Z) = 0.1170 \pm 0.0021(\text{stat+syst})$  that correspond to  $\Lambda_{\overline{MS}}^{(3)} = 337 \pm 29(\text{stat+syst})$  MeV or  $\Lambda_{\overline{MS}}^{(4)} = 301 \pm 30(\text{stat+syst})$  MeV. Pure statistical error of  $\alpha_s(M_Z)$  is 0.0011, which gives only small contribution to the total experimental error. The average bias of the the fitted function against data,

calculated as 
$$\langle (f-y)/\sqrt{\sigma^2 + f(\vec{s})^2} \rangle$$
 is 0.07 that is within its possible statistical fluctuation.

The correlation of  $\alpha_s$  with the HT contribution to  $F_2$  is very large (typical values of correlation coefficients is about -0.9). This means that separation of logarithmic and power effects in the analysis of scaling violation, which is based on the SLAC-BCDMS-NMC data without  $Q^2$  cut, is unstable under various assumptions. In particular, the complete account of point-to-point correlations of the data leads to a shift of the  $\alpha_s$  value by about 3 standard deviations from the results obtained using a simplified statistical inference procedure. Since the HT contribution and the value of  $\alpha_s$  are strongly anticorrelated, the increase of  $\alpha_s$  is accompanied by a decrease of HT. The total effect on the HT magnitude is about a factor of 3/4, as compared with the results of Ref. At the same time  $H_L$  is almost uncorrelated with  $\alpha_s$ , i.e. its value is less model

 $<sup>^{</sup>a}$ We checked that extra polynomial-type factors do not improve the quality of the fits.

<sup>&</sup>lt;sup>b</sup>This effect was also recently observed in the analysis, where  $\alpha_s(M_Z)$  was fixed at 0.120.

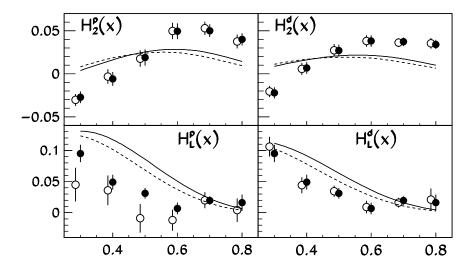


Figure 1: The fitted values of  $H_2(x)$  and  $H_L(x)$  (full circles – the fit with constraint  $H_L^D(x) = H_L^P(x)$ , empty circles – the unconstrainted fit). The curves are predictions of IRR model for  $Q^2 = 2 \text{ GeV}^2$ ,  $n_f = 3$  (full lines) and  $Q^2 = 9 \text{ GeV}^2$ ,  $n_f = 4$  (dashed lines).

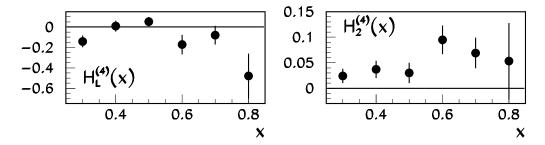


Figure 2: The dependence of twist-6 contributions to  $F_L$  and  $F_2$  on x.

dependent. The predictions of infrared renormalon (IRR) model <sup>8</sup> are also given in Fig. 1. The normalization factor  $A_2'$  was chosen in the universal form:  $A_2' = -\frac{2C_F}{\beta_0} [\Lambda_R]^2 e^{-C}$  where  $C_F = 4/3$ , C = -5/3,  $\beta_0 = 11 - 2/3n_f$ ,  $\Lambda_R = \Lambda_{\overline{MS}}^{(3)} = 337$  MeV, as obtained in our analysis. One can see that the model qualitatively describes the data on  $H_L(x)$  and there is evident discrepancy with the data on  $H_2(x)$ .

We checked how much the analyzed data are sensitive to the twist-6 contribution to  $F_L$  and  $F_2$ . For this purpose we added to  $F_2^{HT}$  or  $F_L^{HT}$  the terms  $H_{L,2}^{(4)}(x)^{\frac{1 \text{ GeV}^4}{Q^4}}$ , where functions  $H_{L,2}^{(4)}(x)$  were the same for proton and deuterium, and parametrized similarly to  $H_{L,2}(x)$ . The fitted values of  $H_{L,2}^{(4)}(x)$  are given in Fig. 2. One can observe the trend to the negative values at highest x for  $H_L^{(4)}(x)$  and trend to the positive values for  $H_2^{(4)}(x)$ . However the statistical significance of the deviation of  $H_{L,2}^{(4)}(x)$  from zero is not very large. In addition, the correlation of  $H_2^{(4)}(x)$  with  $\alpha_s$  and  $H_2(x)$  is extremely strong. Summarizing these observations, we can conclude that the observed deviation of  $H_L^{(4)}(x)$  and  $H_2^{(4)}(x)$  off zero can be considered as qualitative only.

Any renormalization group analysis, which is based on the finite number of perturbative series terms is sensitive to the choice of renormalization scale. This dependence can be used for the rough estimate of the higher orders terms effect that are not accounted for in the analysis; if one includes all terms this dependence should vanish. For estimate of renormalization scale uncertainty in DIS analysis this scale usually is being chosen as  $K_RQ^2$  and the value of  $K_R$ 

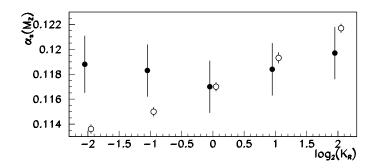


Figure 3: The dependence of  $\alpha_s(M_Z)$  on the renormalization scale. Open circles correspond to the fits with the HT contributions fixed, full circles - to the fits with the HT contributions released.

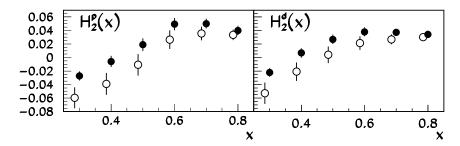


Figure 4: The values of HT contributions for  $K_R = 0.25$  (open circles) and for  $K_R = 1$  (full circles).

is varying in the range of  $0.1-4^{10}$  We studied the dependence of  $\alpha_s$  on the value of  $K_R$  in two cases: for the HT contributions released in the fit and for the HT contributions fixed at the values obtained in the fit with  $K_R = 1$ . The results are given in Fig. 3. One can see that the dependence of  $\alpha_s$  on the renormalization scale is different in the two cases. The reason of this difference is that the  $H_2^{p,d}(x)$  are strongly correlated with  $\alpha_s$ ; when one changes  $K_R$ , the changes in the QCD evolution kernel can be absorbed into the additional power-like contribution. This effect, illustrated in Fig. 4, can be considered as indirect indication that the fitted values of  $H_2^{p,d}(x)$  can include not only genuine power corrections, but also effectively account for higher orders QCD terms. Any way the the value of  $\alpha_s$  is quite stable against renormalization scale variation, if HT are released. At the same time the  $\alpha_s$  errors are significantly larger for this case, which is also a consequence of the large correlation of  $\alpha_s$  with HT, i.e. one can say that the uncertainty in the  $\alpha_s$  value due to the renormalization scale choice is partially hidden in the total experimental error. The spread of the  $\alpha_s$  value in the fits with  $K_R = 0.25-4$  and HT released is 0.0026. With the account of this spread we obtain  $\alpha_s(M_Z) = 0.1183 \pm 0.0021(\exp).\pm0.0013(\text{ren.scale})$ .

### References

- 1. L.W. Whitlow et al., *Phys. Lett.* B **282**, 475 (1992).
- BCDMS collaboration, A.C. Benvenuti et al., Phys. Lett. B 223, 485 (1989);
  BCDMS collaboration, A.C. Benvenuti et al., Phys. Lett. B 237, 592 (1990).
- 3. NM collaboration, M. Arneodo et al., Nucl. Phys. B 483, 3 (1997).
- 4. M. Virchaux, A. Milsztajn, *Phys. Lett.* B **274**, 221 (1992).
- W. Furmanski and R. Petronzio, Z. Phys. C 11, 293 (1982); Phys. Lett. B 97, 437 (1980);
  G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. B 175, 27 (1980);
- 6. S.I. Alekhin, hep-ph/9611213, 1996, to appear in Eur. Phys. Jour.
- 7. H. Georgi, H.D. Politzer, *Phys. Rev.* D **14**, 1829 (1976).
- 8. Yu.L. Dokshitzer, G. Marchesini, B.R. Webber, Nucl. Phys. B 469, 93 (1996); E. Stein, M. Meyer-Hermann, A. Schäfer, L. Mankiewicz, Phys. Lett. B 376, 177 (1996);

- M. Dasgupta, B.R. Webber, *Phys. Lett.* B **382**, 273 (1996).
- 9. U.K. Yang, A. Bodek, Report No. UR-1543, ER-40685-929, hep-ph/9809480.
- 10. A.D. Martin, R.G. Roberts, and W.J. Stirling, *Phys. Lett. B* **266**, 273 (1991).